Residue Cycles of Fib(mod *m*)

an Introduction by Jonathan Haek

A (general) **Fibonacci** series is an infinite sequence of numbers generated by the following recursive relation:

$$F_n = F_{n-1} + F_{n-2} \quad \text{where } a = F_0 \text{ is the first integer in series,}$$
$$\text{and } b = F_1 \text{ is the second integer in series}$$

When $a = 0$ and $b = 1$, the sequence of numbers constitutes THE Fibonacci numbers:

$$F_n = 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, \ldots$$

When $a = 2$ and $b = 1$, the sequence of numbers constitutes the Lucas numbers:

$$L_n = 2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, \ldots$$

While a sequence of Fibonacci numbers is not itself useful musically, due to the rapid growth toward extremely large numbers [e.g., F_{100} = 354224848179261915075], taking the **modulus** of each number in a Fibonacci sequence [i.e., the remainder, or residue, of each Fibonacci number divided by a particular integer, *m*, the modulus – abbreviated, Fib(mod *m*)] yields potentially useful information.

What happens when you take Fib(mod *m*)?

Taking the residue of each integer (mod *m*) in a sequence of Fibonacci numbers forms a class of integers [similar to pitch class, where many different pitches (in Hz, for instance) form a class under octave equivalency (multiplication)] which is unique to each *m*. These residue classes are peculiar in themselves because many residue classes associated with Fib(mod *m*) **do not contain every integer** from 0 to *m*-1 as do the residue classes associated with the set of all integers modulus *m* [e.g., the residue class of F_n(mod 110) contains only 24 of the 110 possible residues]. Probably more importantly, however, is that successive integers of Fib(mod *m*) create a repeating cycle of integers within the residue class. Every residue cycle is a finite, **ordered series** of integers of varying length, and is able to be divided many ways into **sections** which are multiples of each other. These (and a host of other) peculiar qualities and properties of the residue classes and cycles exhibit high potential for developing new serial composition techniques using residue cycles of Fib(mod *m*).